B.Sc. Part II 4th Paper

Vectors (continued)

Q. If $\vec{f} = (x+y+1)\vec{i} + \vec{j} + (-x-y)\vec{k}$, find $\text{curl}\,\vec{f}$ and $\vec{f}\cdot\text{curl}\,\vec{f}$.

Soln.

$$\text{curl}\,\vec{f} = \nabla \times \vec{f}$$

$$\Rightarrow \text{curl}\,\vec{f} = \left(\vec{i}\frac{\partial}{\partial x} + \vec{j}\frac{\partial}{\partial y} + \vec{k}\frac{\partial}{\partial z}\right) \times \left[(x+y+1)\vec{i} + \vec{j} + (-x-y)\vec{k}\right]$$

$$\Rightarrow \text{curl}\,\vec{f} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y+1 & 1 & -x-y \end{vmatrix}$$

$$\Rightarrow \text{curl}\,\vec{f} = \vec{i}\left[\frac{\partial}{\partial y}(-x-y) - \frac{\partial}{\partial z}(1)\right]$$

$$-\vec{j}\left[\frac{\partial}{\partial x}(-x-y) - \frac{\partial}{\partial z}(x+y+1)\right]$$

$$+\vec{k}\left[\frac{\partial}{\partial x}(1) - \frac{\partial}{\partial y}(x+y+1)\right]$$

$$\Rightarrow \text{curl}\,\vec{f} = \vec{i}\left[-1-0\right] - \vec{j}\left[-1-0\right] + \vec{k}\left[0-1\right]$$

$\Rightarrow \quad \text{curl} \vec{f} = -\vec{i} + \vec{j} - \vec{k}$

Now $\vec{f} \cdot \text{curl} \vec{f} = \left[(x+y+1)\vec{i} + \vec{j} + (-x-y)\vec{k}\right] \cdot \left[-\vec{i} + \vec{j} - \vec{k}\right]$

$\Rightarrow \vec{f} \cdot \text{curl} \vec{f} = (x+y+1)\cdot(-1) + 1\cdot 1 + (-x-y)\cdot(-1)$

$= \cancel{-x} \cancel{-y} \cancel{-1} \cancel{+1} \cancel{+x} \cancel{+y} = 0$

So, $\vec{f} \cdot \text{curl} \vec{f} = \underline{\underline{0}}$

<u>Formula</u> $\text{curl} \vec{f} = \nabla \times \vec{f}$

$= \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}\right) \times (\vec{f})$

If $\vec{f} = x\vec{i} + y\vec{j} + z\vec{k}$ then

$\text{curl} \vec{f} = \left(\vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}\right) \times (x\vec{i} + y\vec{j} + z\vec{k})$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$